Georgia Tech

The Atlanta Journal-Constitution

Creating software to compute the optimal number of newspapers to deliver to each sales outlet

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Overview

Current Model:

Maximizing Circulation

Optimization Model:

Maximizing Profit

Historical Sales Data → Optimization Model → Optimal Draw → Maximized Profit

Additional profit per year : $1.8 million

Optimization Model Inputs

- Marginal Revenue
- Marginal Cost
- Salvage Revenue
- Demand Distributions
- Theft
- Shrinkage

Optimization Model

Outlet Types

Theft Outlet

Shrinkage Outlet

Non-Theft/
Non-Shrinkage Outlet

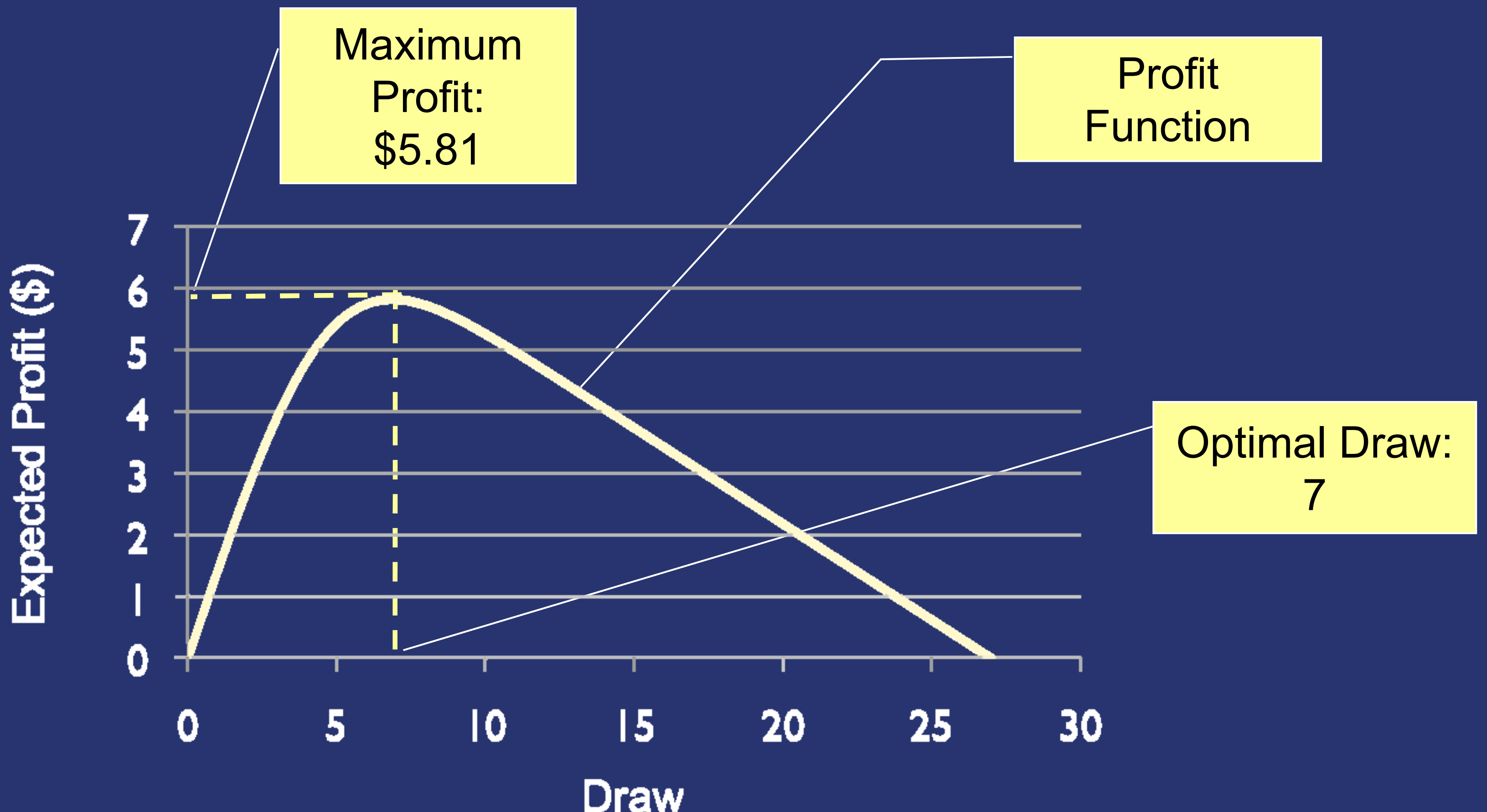
Profit Function

Case without shrinkage or theft:

$$E_D[g(x,D)] = \boxed{\sum_{d=0}^{x} P[D=d]}\left(\underline{rd} + \underline{s(x-d)}\right) + \boxed{\left(1 - \sum_{d=0}^{x} P[D=d]\right)}\left(\underline{rx}\right) - \underline{cx}$$

x	=	Decision variable, draw	r	=	Marginal revenue
D	=	Demand	c	=	Marginal cost
			s	=	Salvage revenue

Profit Function

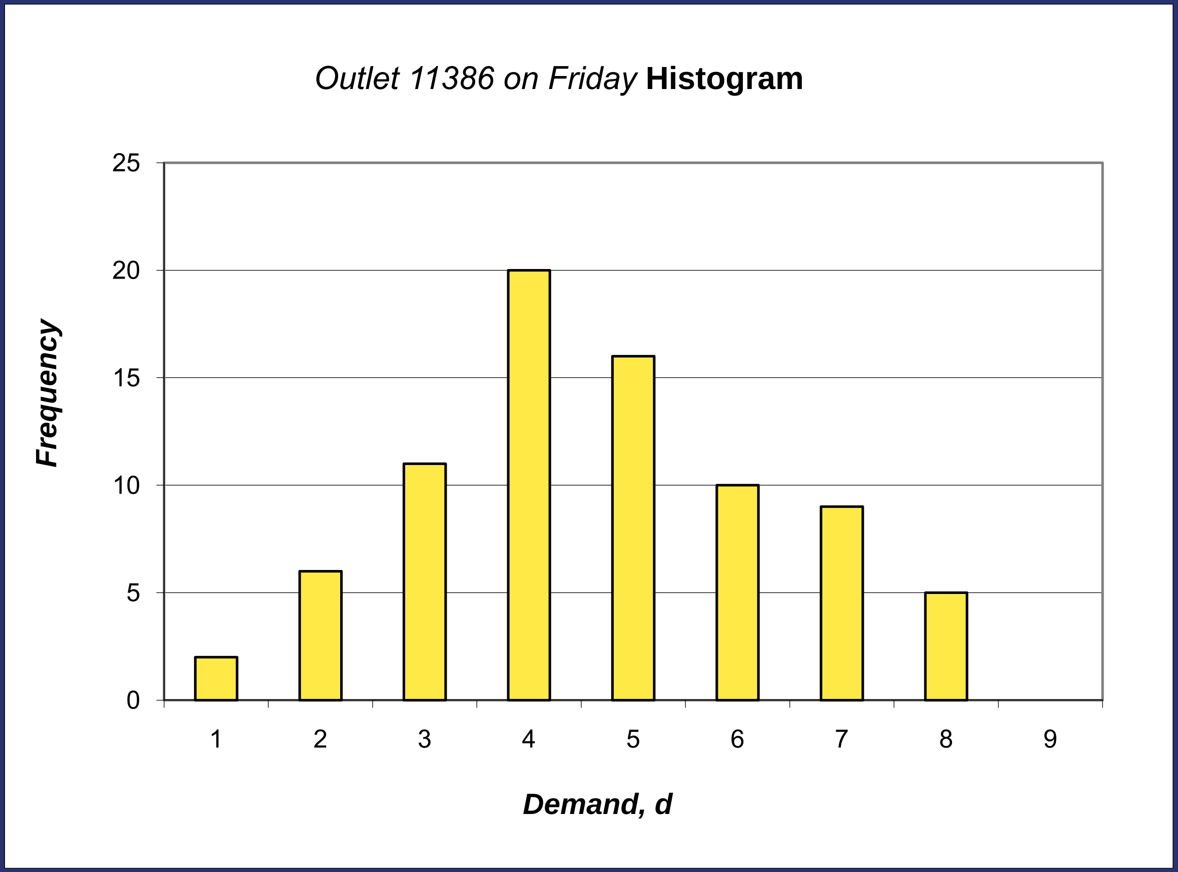


Optimization Model Inputs

- Marginal Revenue
- Marginal Cost
- Salvage Revenue
- Demand Distributions
- Theft
- Shrinkage

Optimization Model

Demand Distributions



Demand Distributions

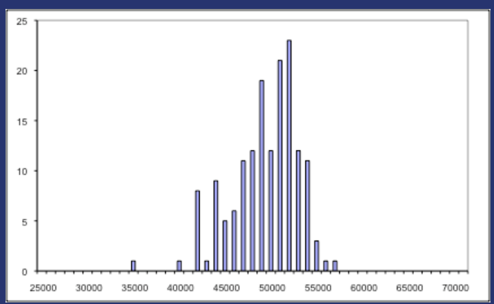
- Censored data
- Shifting sales trends

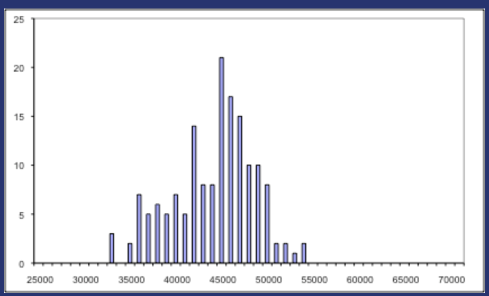
July 23, 2007 – Jan. 22, 2008

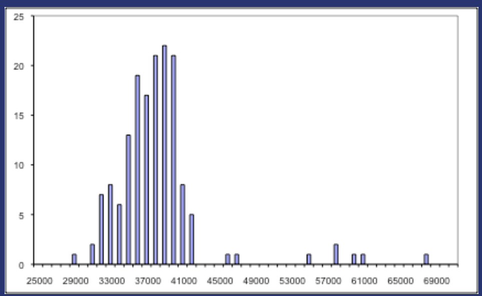
Jan. 23, 2008 – July 24, 2008

July 25, 2008 – Jan. 29, 2009

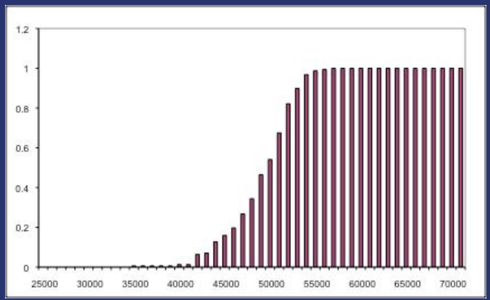
Histograms

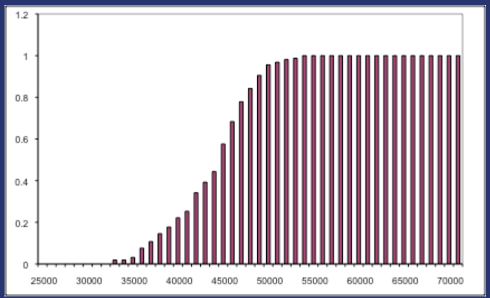


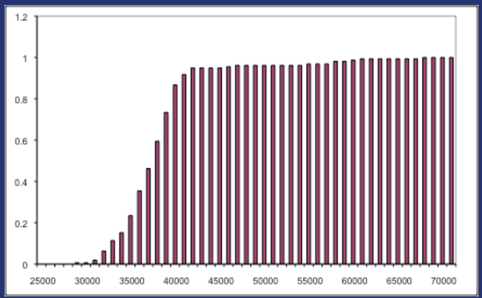




CDFs







Covariates

Covariates tested:

Weather	4th of July
Unemployment Rate	Tax Free Holiday
Seasonality	Labor Day
Price	Wednesday before Thanksgiving
Time	Thanksgiving
New Year's Day	Thanksgiving Weekend
Martin Luther King Day	Christmas Eve
Memorial Day	Christmas Day

Covariates

Covariates with greatest impact:

- Weather
- ✓ **Unemployment Rate**
- Seasonality
- ✓ **Price**
- ✓ **Time**
- ✓ **New Year's Day**
- Martin Luther King Day
- Memorial Day
- 4th of July
- Tax Free Holiday
- ✓ **Labor Day**
- Wednesday before Thanksgiving
- ✓ **Thanksgiving**
- Thanksgiving weekend
- ✓ **Christmas Eve**
- ✓ **Christmas Day**

Demand Distributions

- Demand distributions considered:
  - Exponential
  - Gaussian
  - Log-Logistic
  - Log-Normal
  - Logistic
  - Poisson
  - Weibull
- Maximum Likelihood Estimation (MLE)

Maximum Likelihood Estimation

MLE example for Poisson distribution without theft:

$$LL(\beta) = \sum_{j=1}^{n} \left[\left(1 - z_j\right) \log\left(\frac{e^{-\lambda_j} \left(\lambda_j\right)^{k_j}}{k_j!} \right) + z_j \log\left(1 - \sum_{l=0}^{k_j-1} \frac{e^{-\lambda_j} \left(\lambda_j\right)^{l}}{l!} \right) \right]$$

λ_j	=	$\beta_0 + \beta_1 x_1 + \ldots \beta_m x_{mj}$
β_0 - β_m	=	Parameters to be estimated
x_1 - x_m	=	Covariate values
k_j	=	Sold quantity of data point *j*
z_j	=	1 if data point *j* is censored; 0 otherwise

Maximum Likelihood Estimation

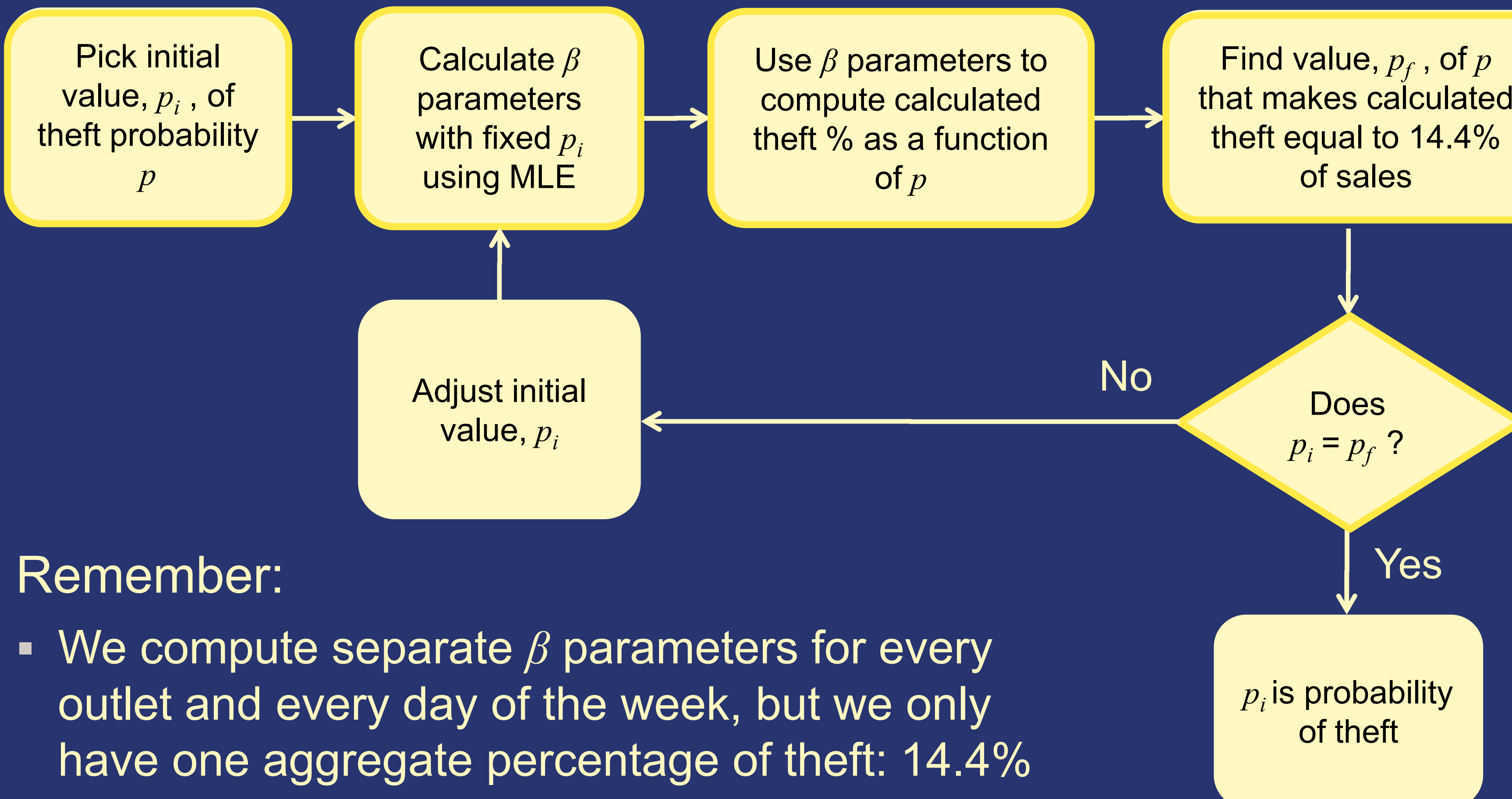
MLE example for Poisson distribution with theft:

$$LL(\beta) = \sum_{j=1}^{n}\left[\left(1-z_j\right)\log\left(\frac{e^{-\lambda_j}\left(\lambda_j\right)^{k_j}\left(1-p\right)^{k_j}}{k_j!}\right)\right.$$

$$\left.+z_j\log\left[\left(1-\sum_{l=0}^{k_{j-1}}\frac{e^{-\lambda_j}\left(\lambda_j\right)^{l}\left(1-p\right)^{k_{j-1}}}{l!}\right)+\sum_{l=1}^{k_{j-1}}\left(\frac{e^{-\lambda_j}\left(\lambda_j\right)^{l}}{l!}\left(1-\left(1-p\right)^{l}\right)\right)\right]\right]$$

λ_j	=	$\beta_0+\beta_1 x_1+\ldots\beta_m x_{mj}$
β_0 - β_m	=	Parameters to be estimated
x_1 - x_m	=	Covariate values
k_j	=	Sold quantity of data point *j*
z_j	=	1 if data point *j* is censored; 0 otherwise
p	=	Probability of theft

Probability of Theft

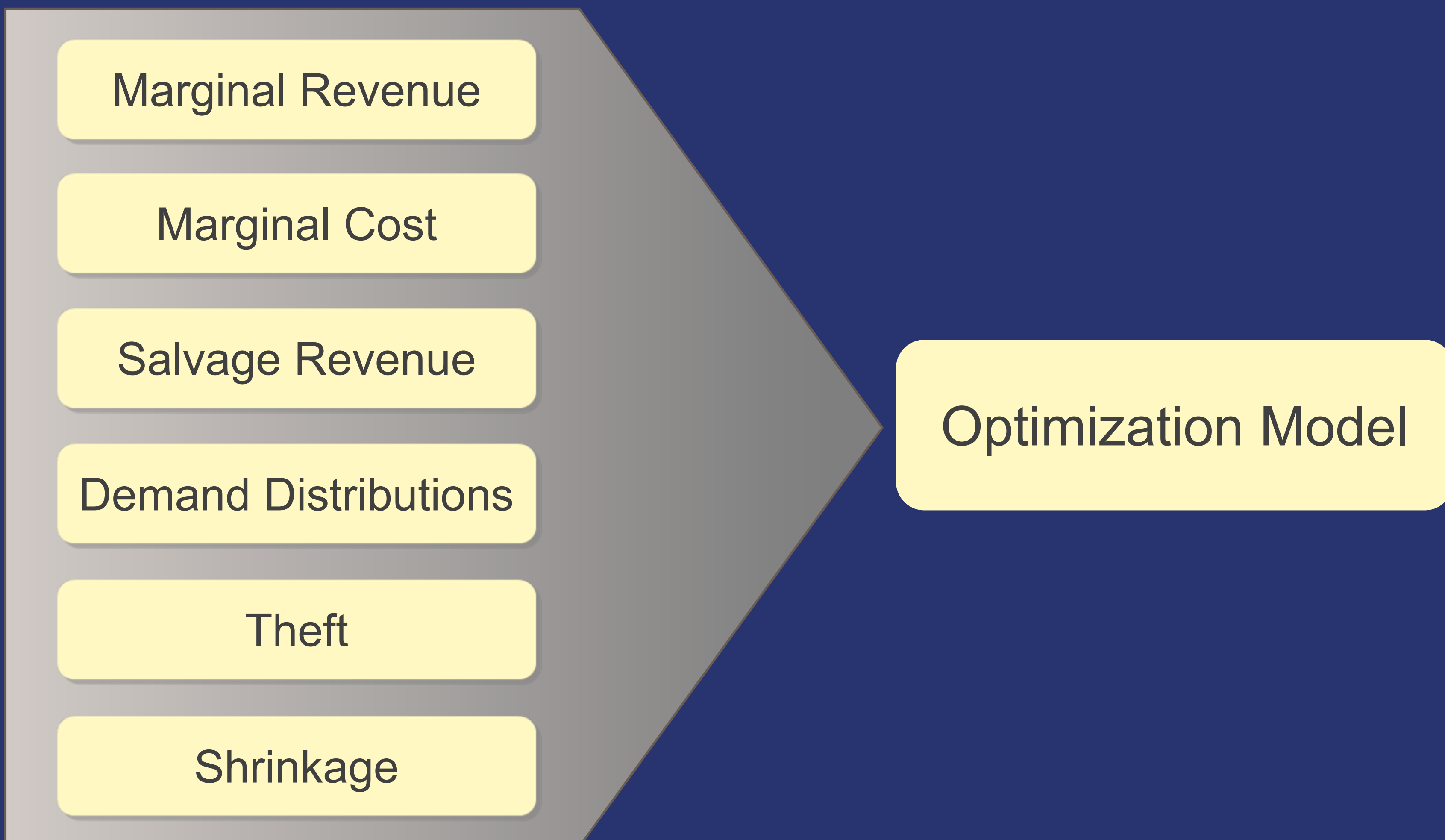


- Remember:
  - We compute separate β parameters for every outlet and every day of the week, but we only have one aggregate percentage of theft: 14.4%

Demand Distributions

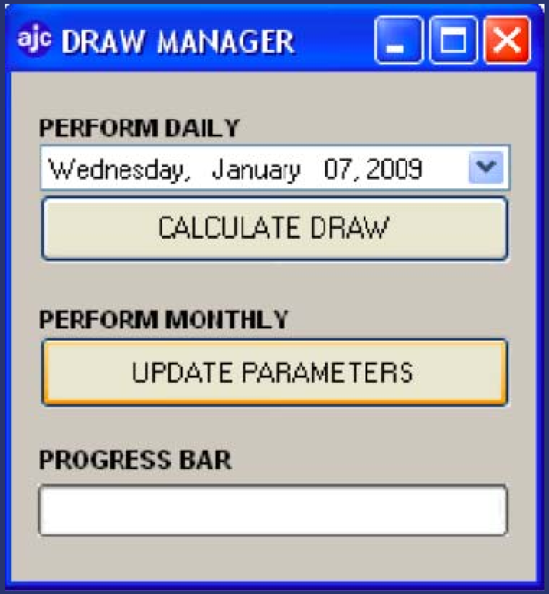
- Demand distributions tested:
  - Exponential
  - Gaussian
  - Log-Logistic
  - Log-Normal
  - Logistic
  - Poisson
  - Weibull
- Poisson distribution

Optimization Model Inputs

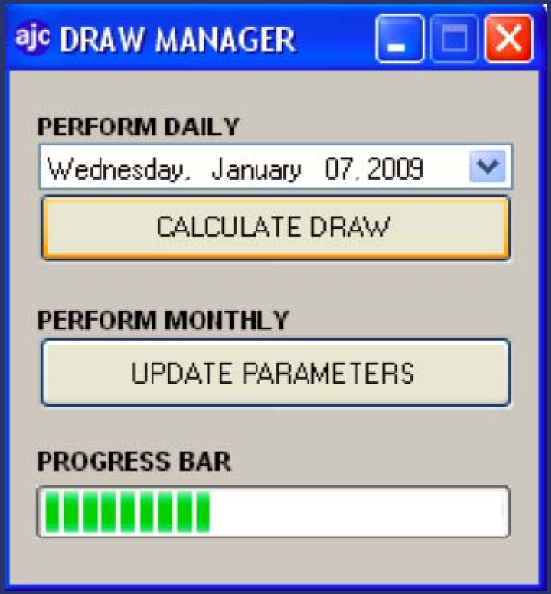


Software Snapshot

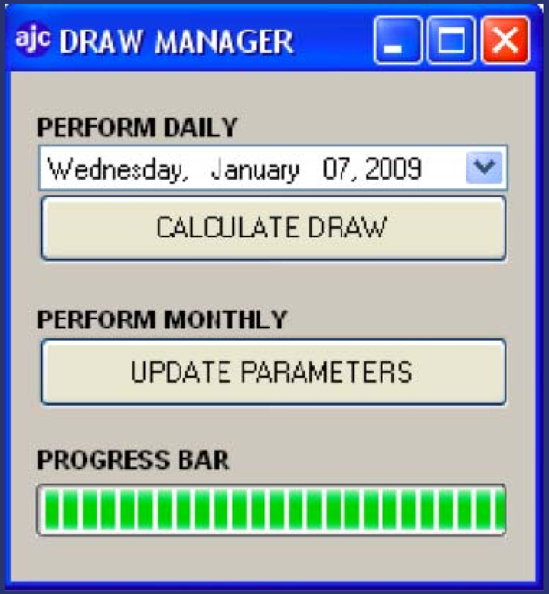
Start



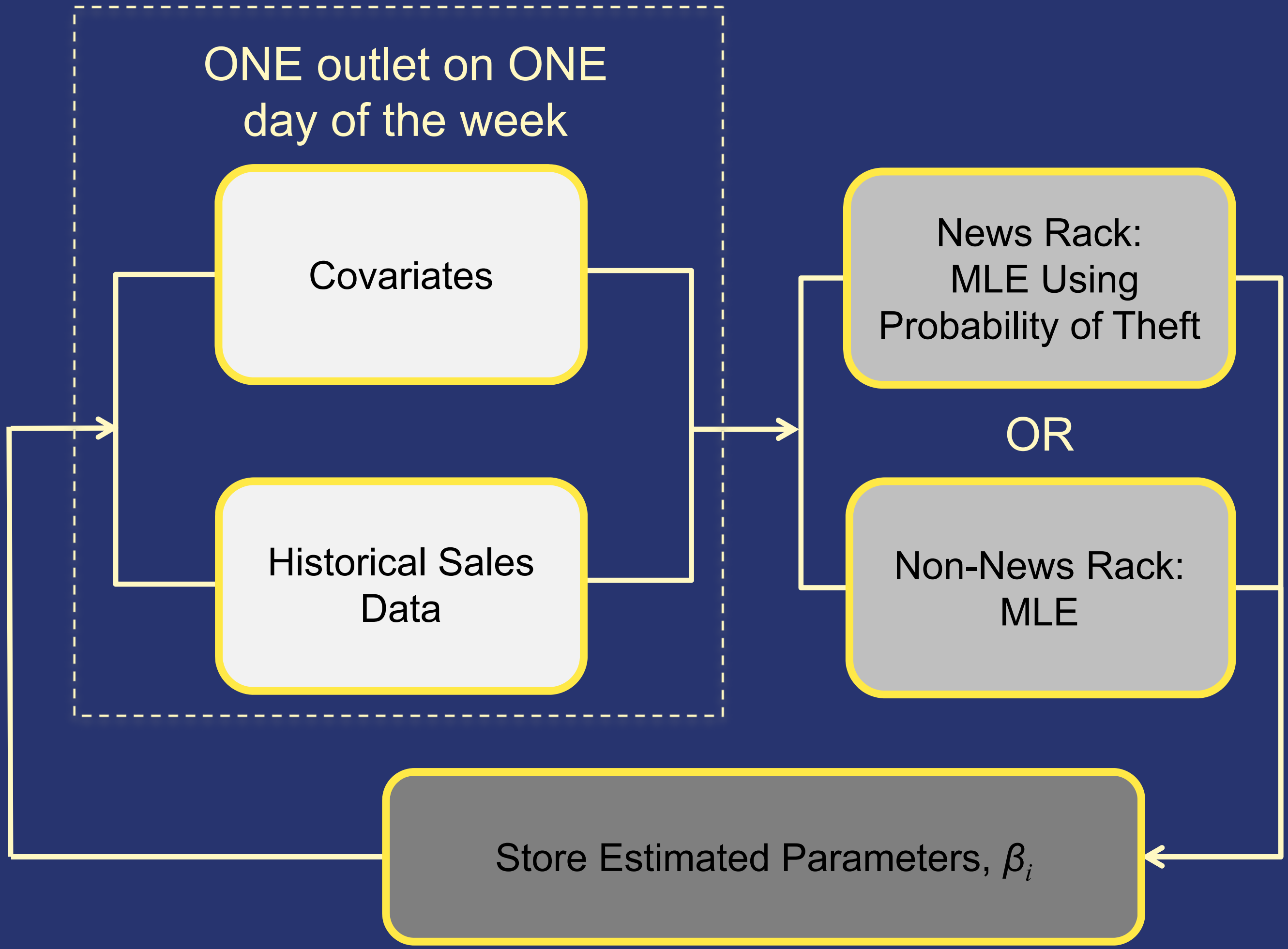
In Progress



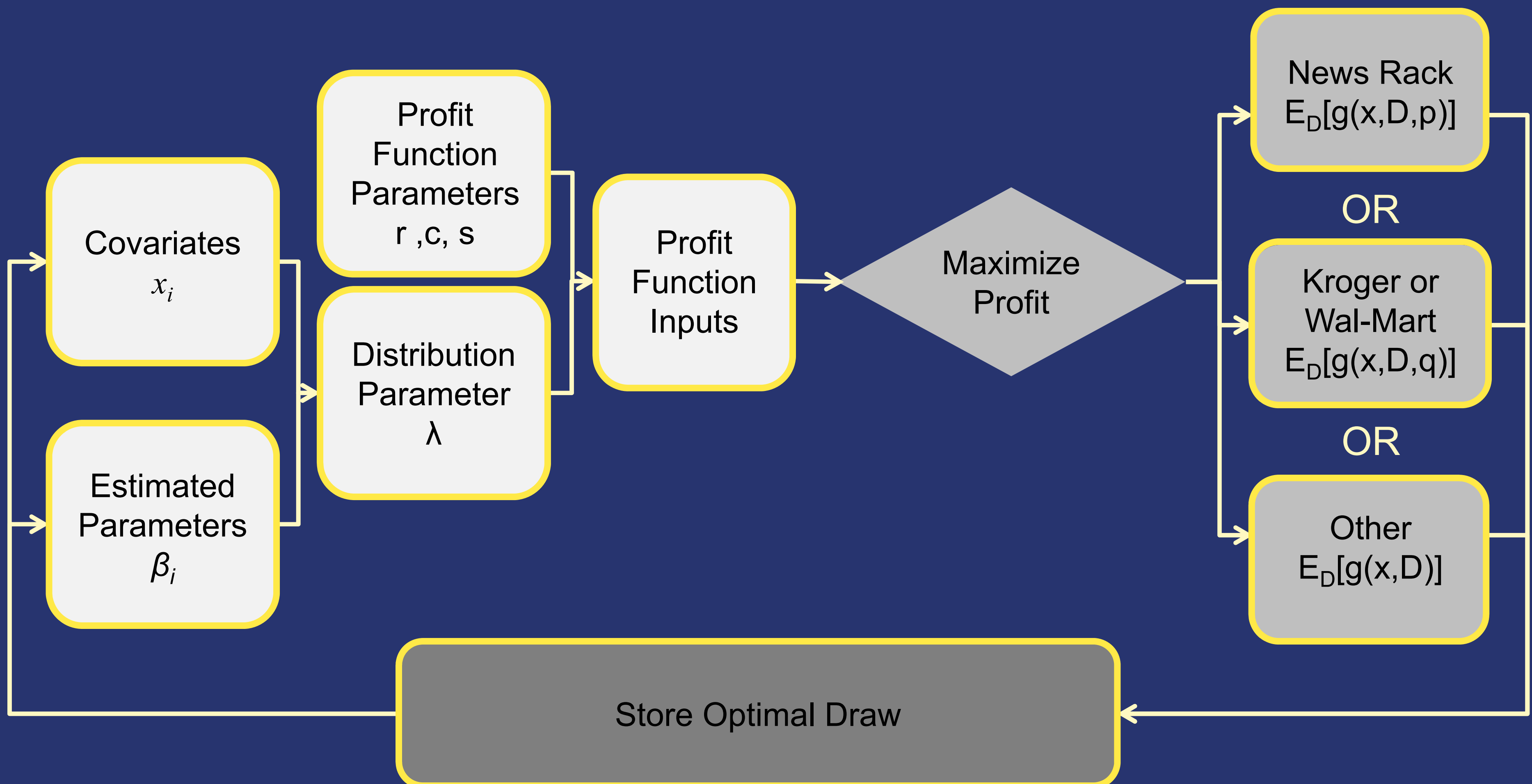
Complete



Update Parameters



Calculate Draw



Validation

- Optimization model data:
  - July 2007 – November 2008
- Validation:
  - December 2008
- Assumptions:
  - Calculate AJC estimated profit for comparison

Validation

Sample of results:

Outlet ID	GT Draw	GT Profit	AJC Draw	AJC Profit	Additional Profit	Sold Quantity
4820	7	$2.98	6	$2.89	$0.09	6
1539	2	$0.19	4	$0.06	$0.13	2

GT profit versus AJC profit summary:

Compare GT & AJC	GT Profit	AJC Profit
Per outlet per day:	$6.04	$5.28
All outlets per day:	$45,500	$39,700
All outlets per year:	$14.2 million	$12.4 million

Sensitivity Analysis

What if we used the Poisson distribution to calculate the draws, but the demand is actually Gaussian distributed?

Comparison	Expected Profit
AJC Profit	\$12.5 million
GT Profit using Poisson draws	\$14.45 million
GT Profit using Gaussian draws	\$14.47 million

Sensitivity Analysis

Include 100% of advertising:

Comparison	Expected Profit
AJC Profit	$12.4 million
GT Profit	$14.2 million

Additional Profit → $1.8 million

Include 50% of advertising:

Comparison	Expected Profit
AJC Profit	$7.8 million
GT Profit	$8.7 million

Additional Profit → $900,000

Include 0% of advertising:

Comparison	Expected Profit
AJC Profit	$3.1 million
GT Profit	$3.55 million

Additional Profit → $450,000

Thank You

Client Advisor: Mike Burlingame
Senior Director, Consumer Sales & Retention
The Atlanta Journal-Constitution

Faculty Advisor: Dr. Anton Kleywegt

Appendices

- Appendix I: Objective Profit Function (Shrinkage)
- Appendix II: Objective Profit Function (Theft)
- Appendix III: Profit Function (Theft) Comparisons
- Appendix IV: Fixed Point Calculation

Objective Function

Case with shrinkage:

$$E_D[g(x,D,q)] = \boxed{\sum_{d=0}^{x} P[D=d]}\left(\underline{r_{adv}d} + \underline{(1-q)r_{sales}d} + s(x-d)\right)$$
$$+\boxed{\left(1-\sum_{d=0}^{x} P[D=d]\right)}\left(\underline{r_{adv}x} + \underline{(1-q)r_{sales}x}\right) - cx$$

x	=	Decision variable, draw	r_{adv}	=	Marginal advertising revenue
D	=	Demand	r_{sales}	=	Marginal sales revenue
q	=	Probability of shrinkage	c	=	Marginal cost
			s	=	Salvage revenue

Objective Function

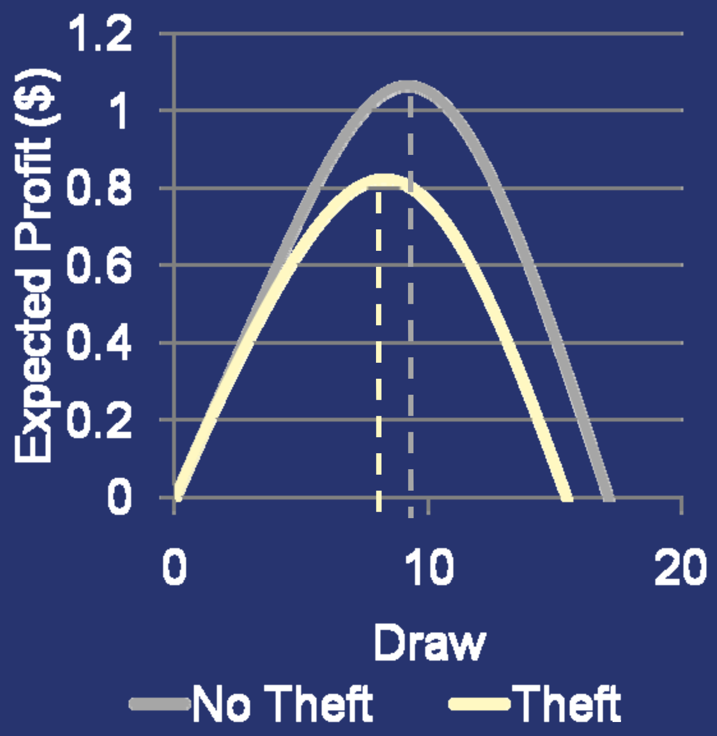
Case with theft:

$$
\begin{aligned}
E_D[g(x,D,p)] = {} & \boxed{\sum_{d=1}^{x} P(D=d)} \left[\sum_{j=1}^{d} \underline{(1-p)^{j-1} p(r_{sales} j + r_{adv} x)} + \underline{(1-p)^{d} (r_{sales} d + r_{adv} d} + \underline{s(x-d})) \right] \\
& + \boxed{\left(1 - \sum_{d=0}^{x} P(D=d) \right)} \left[\sum_{j=1}^{x} \underline{(1-p)^{j-1} p(r_{sales} j + r_{adv} x) + (1-p)^{x} (r_{sales} x + r_{adv} x)} \right] \\
& + \underline{\left[P(D=0)(sx) \right]} - \underline{cx}
\end{aligned}
$$

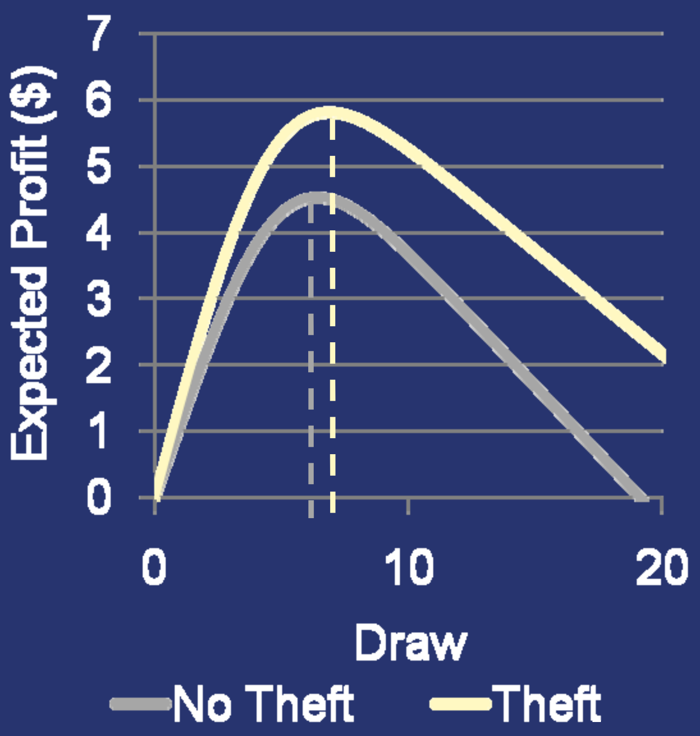
x	=	Decision variable, draw
D	=	Demand
p	=	Probability of theft
r_{adv}	=	Marginal advertising revenue
r_{sales}	=	Marginal sales revenue
c	=	Marginal cost
s	=	Salvage revenue

Profit Function

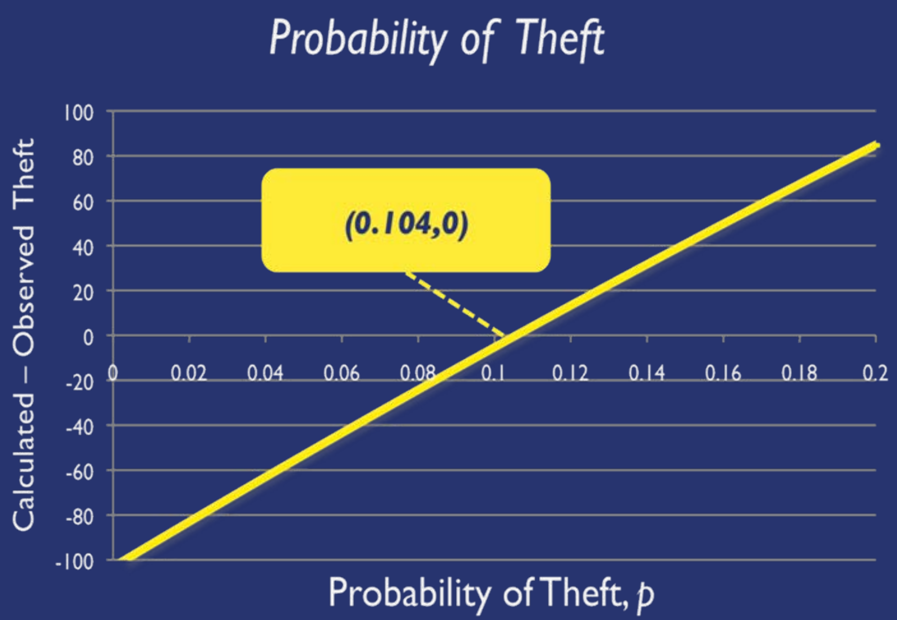
Example 1:
Marginal advertising revenue is low

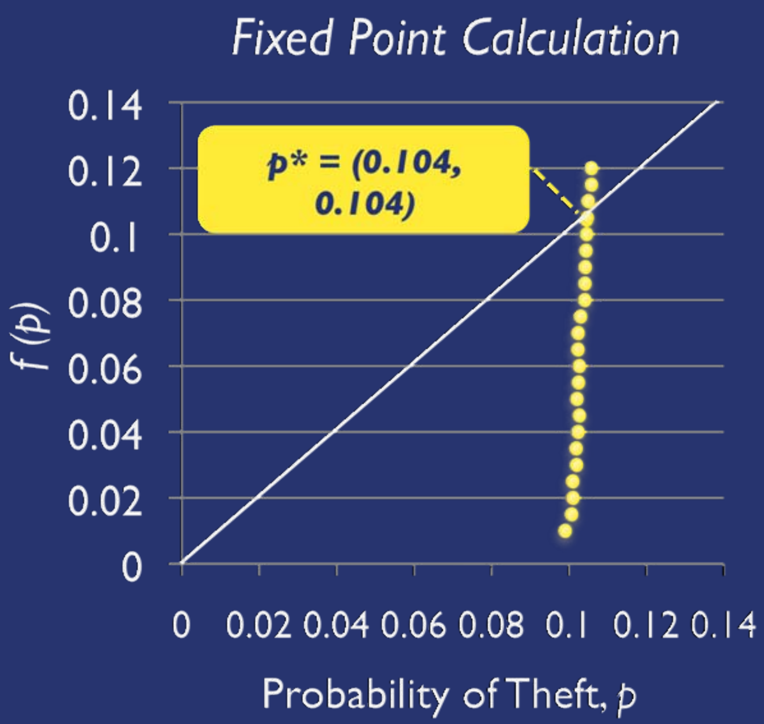


Example 2:
Marginal advertising revenue is high



Probability of Theft





$$\sum_{\text{Data Set } i} \left[\sum_{d=1}^{x_i} P_i(D=d) \sum_{j=1}^{d} (1-p)^{j-1} p(x_i - j) + \left(1 - \sum_{d=0}^{x_i} P_i(D=d)\right) \sum_{j=1}^{x_i} (1-p)^{j-1} p(x_i - j) \right] = 14.4\% \text{ of Total Sales}$$